Application of Total Energy Control for High-Performance Aircraft Vertical Transitions

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Energy management guidance algorithms for automated, fast transitions between cruise states are presented. These algorithms use total energy control principles to regulate throttle and flight control inputs. Two principal algorithms are used to perform cruise state transitions. 1) Energy hold/altitude hold: This algorithm consists of two guidance laws that regulate thrust and vertical lift to acquire and maintain a desired height/energy cruise point. 2) Energy capture: This algorithm regulates flight-path angle at fixed throttle to coordinate height/speed transitions to a desired cruise point. It is primarily used for large energy transitions at maximum or minimum throttle settings, and requires switching to energy hold/altitude hold as the cruise state is approached. Algorithm performance is illustrated using five-degree-of-freedom simulations of a modern, high-performance fighter aircraft.

Nomenclature

C_D	= coefficient of drag
C_L	= coefficient of lift
$C_{l\max}$	= coefficient of lift at maximum angle of attack
D	= drag, lb
E	= total energy per unit weight, $= h + V^2/2g$
G-lim	= structural g limit, g
g	= acceleration of gravity, f/s^2
h	= altitude, ft
K_0, K_1, K_2, K_3	= energy hold control gains
k_h, k_γ, k_η	= altitude hold and flight-path control gains
L	= lift normal to the velocity axis, lb
M	= Mach number
ps	= excess power, ft/s
Q	= dynamic pressure, lb/ft^2
S	= wing reference area, ft^2
T	= total thrust, lb
t_f	= time to capture the final states h_f and V_f
V	= velocity, ft/s
W	= weight, lb
x, y	= horizontal plane Cartesian coordinates, ft
α	= angle of attack, rad
γ	= flight-path angle, rad
ΔE	= energy to go, $= E_f - E$
Δh	= height to go, ft
ΔV	= velocity to go, ft/s
$\Delta\psi$	= heading to go, rad
η	= normal load factor, $= L/W$
η_H	= horizontal load factor, $= \eta \sin\phi$
η_V	= vertical load factor, $= \eta \cos\phi$
τ	= engine time constant, s
τ_v	= vertical load factor lag, s
ϕ	= bank angle or roll, rad
ψ	= horizontal axis heading angle, rad

Subscripts

c	= output commands from guidance laws
f	= desired final values (end states)
min, max	= minimum and maximum limits, respectively
r	= reference values

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Introduction

IN next-generation tactical aircraft, much greater flight-path automation is desired to reduce pilot work load and to implement coordinated attack and defense tactics in beyond-visual-range air-to-air engagements. This research work over a several-year span is based on a two-layer concept for flight-path automation. On the top layer resides the mission/tactics software manager. The mission/tactics manager is conceptually a rule-based system that selects an appropriate flight mode and tactics based on the overall situation and the pilot's mission objectives. On the bottom layer are the guidance laws used to implement the desired flight modes and engagement tactics. These guidance modes translate high-level flight control and propulsion control inputs to the vehicle management system. The subject of this paper is a guidance law to perform fast (near minimum time) vertical plane/speed transitions to a desired cruise point. This guidance law can be combined with an appropriate horizontal plane steering law to implement air combat engagement tactics. For example, the vertical guidance laws could be used with horizontal plane intercept steering to transition from high-altitude, maximum-duration flight to medium-altitude, supersonic flight for a medium-range missile engagement.

The initial work in developing vertical plane guidance laws was strongly influenced by previous papers that used singular perturbation theory (SPT) for various energy management and target interception problems.[1-4] These papers suggest the decomposition of the aircraft vertical dynamics into slow and fast modes, with specific energy as the slowest mode and height and flight-path angle as the fastest mode dynamics. Decomposition leads to decoupled controls, with thrust controlling the energy state and load factor controlling the height and flight-path angle dynamics. The energy hold/altitude hold guidance algorithm is based on this concept and specifically uses a linearized feedback controller for regulating height and flight-path angle.[3]

It was found during algorithm testing with a high-performance fighter model that the energy hold and altitude hold guidance laws are generally suboptimal for fast vertical state transitions such that the throttle is saturated at minimum or maximum thrust levels until the desired energy state is approached. The problem is that the height and speed transitions are uncoordinated. Even with load-factor limiting to prevent excessive losses in energy, the altitude controller predominates, and these transitions usually end with a constant altitude acceleration or deceleration to attain the desired cruise speed. Attempts were made to introduce a reference path in height/energy space and to follow this path using the altitude hold controller to coordinate height and speed capture. This approach did not yield faster transitions because coupling between the altitude and energy states often resulted in energy state overshoots, i.e., energy capture was not attained until after altitude capture was achieved.

The problem of coordinating height and speed for fast state transitions was solved in a simple and elegant manner by using total energy control (TEC) principles.[5,6] The core ideas are to use throttle to control total energy and to use flight-path angle to control the distribution of energy between height and speed requirements. This paper introduces one additional principle, namely, to manage flight-path angle such that time-to-altitude capture equals time-to-energy capture. This leads to an implicit equation for flight-path angle, which is the basis for the energy capture algorithm. Fast state transitions are obtained by using the energy capture algorithm to acquire and follow the desired flight-path angle until commanded thrust backs off from minimum or maximum saturation levels and then to use the altitude hold/energy hold guidance laws. In other words, the energy capture law sets up the right conditions for decoupled regulation of thrust and load factor to steady-state flight.

From a systems standpoint, the guidance algorithms described here are generic and can be added to existing outer-loop flight management systems. This permits design and implementation of advanced flight management systems independent of the flight control and propulsion control systems. In fact, these algorithms have been demonstrated in closed-loop flight simulation for a high fidelity, six-degree-of-freedom (DOF) model of an existing F-15 aircraft. Additional development is required, however, to properly account for vertical winds and temperature variations.

Altitude/Energy Hold Guidance

Aircraft Model Dynamics

The guidance laws are based on a simple four-state, point-mass model of the aircraft vertical plane dynamics[2]

$$\dot{E} = [T - D(\eta)]V/W \tag{1}$$

$$\dot{T} = (T_c - T)/\tau \tag{2}$$

$$\dot{h} = V \cdot \sin\gamma \tag{3}$$

$$\dot{\gamma} = (g/V) \cdot (\eta_v - \cos\gamma) \tag{4}$$

where total load factor η is determined from horizontal and vertical components η_H and η_V:

$$\eta = [(\eta_H) + (\eta_V)^2]^{1/2} \tag{5}$$

These equations assume small angles of attack and thrust along the longitudinal axis. In these equations, η_H is an input quantity obtained from a separate horizontal plane guidance law. The controls are vertical load factor η_V and thrust command T_c. The altitude hold and energy hold algorithms are only appropriate for use in the interior of the flight envelope so that state constraints can be ignored. The controls constraints are of the form

$$|\eta| \leq \min\,(G\text{-lim},\ QS\,C_{l\max}/W) \tag{6}$$

$$T_{\min}\,(h, M) \leq T_c \leq T_{\max}\,(h, M) \tag{7}$$

where $C_{l\max}$ denotes the coefficient of lift at maximum angle of attack as a function of Mach, and $T_{\min}$ and T_{max} denote minimum and maximum (afterburner) thrust tables, respectively vs height and Mach.

Finally, drag D in Eq. (1) is an implicit function of height, Mach, and η computed using C_D tables vs C_L and Mach

$$D = Q\,S\,C_D\,(C_L, M) \tag{8}$$

$$C_L = \eta W/QS \tag{9}$$

Total energy E is used as a state variable rather than speed V since there is a natural decoupling of the slow energy states E and T and the fast vertical motion states h and γ. The relationships between E and V are derived easily from the energy state equation

$$E = h + V^2/2g \tag{10}$$

Solving Eq. (10) for V yields

$$V = [2g(E - h)]^{1/2} \tag{11}$$

Similarly, differentiating Eq. (10) yields an explicit relationship between $\dot{E}$ and $\dot{V}$

$$\dot{E} = \dot{h} + V\,\dot{V}/g$$

$$= V\,(\sin\gamma + \dot{V}/g) \tag{12}$$

Although the simplified point mass equations [Eqs. (1–4)] are used to derive the vertical plane guidance laws, more complex equations are used to evaluate performance. Moreover, the inputs to the guidance laws must be physically measurable quantities. Equations (10) and (12) may be used in practice to determine the energy and energy rate inputs.

Energy Hold Throttle Control Law

The idea behind the throttle control law is to solve for thrust command such that $\dot{E} = -K(E_f - E)$. From Eq. (1), we then obtain the prototype control law

$$T_c = (W/V)\,K\,(E - E_f) + D(\eta) \tag{13}$$

The prototype control law cannot be used directly since the actual drag is not known. However, it suggests the use of two terms to control thrust: 1) a linear feedback law depending on $\Delta E = E_f - E$ and $\dot{E}$ for control damping, and 2) an estimate of predicted drag based on commanded load factor and dynamic pressure, i.e.,

$$D(\eta) = Q\,S\,C_D\,[W\eta/(QS), M] \tag{14}$$

The energy hold throttle law that gave best overall performance in simulations is shown in Fig. 1. The addition of estimated drag is essential for anticipation of load-factor induced energy transients. An integrator is then needed in the control law to eliminate bias errors due to differences between estimated and actual drag. It was found after several design trials that a feedback loop around the integrator was also needed to avoid high gains. Ignoring the height and flight-path dynamics, the overall closed-loop feedback system for energy

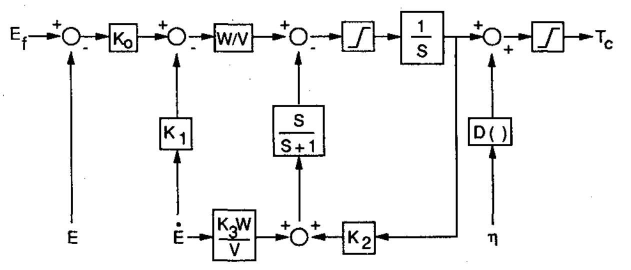

Fig. 1 Energy hold thrust control flow diagram.

is a fourth-order system. Thus, with the four gains $K_0, \ldots, K_3$, the closed-loop poles of this control system can be assigned to obtain fast response characteristics with sufficient damping to prevent excessive overshoots.

When thrust command reaches the saturation bounds $T_{\min}$ or $T_{\max}$, the integrator state is limited such that T_c exactly equals the saturation bound after the $D(\eta)$ summation shown in Fig. 1. This allows the control law to respond quickly when backing off the saturation limits.

Altitude Hold Load Factor Control

The guidance law for tracking the desired final altitude h_f is based on feedback of flight-path angle and altitude error

$$\eta_V = 1 + k_h (h_f - h) - k_\gamma \gamma \tag{15}$$

The gain constants k_h and k_γ are chosen to have specified closed-loop damping and bandwidth. It was known from previous studies[7] that the guidance law [Eq. (15)] can be used even for very large altitude transitions, provided that the closed-loop poles are scheduled as functions of $|h_f - h|$.

Once the linearized load factor n_V is obtained from Eq. (15), η is computed from Eq. (5) and is then limited based on minimum and maximum limits

$$\eta_{\min} \leq |\eta| \leq \eta_{\max} \tag{16}$$

The minimum limit $\eta_{\min}$ is used to stabilize the bank angle command for rolling the aircraft. It is implemented as a dead-band limiter with magnitude = 0.25 g. The maximum limit $\eta_{\max}$ must be consistent with the physical limits [Eq. (6)], but is normally determined by energy management constraints of the form

$$\dot{E}\,(T_{\max}, \eta_{\max}) = f\,(\Delta E) \tag{17}$$

where $f(\Delta E)$ varies between $-ps$ for $\Delta E < -2000$ ft and $+ps/2$ for $\Delta E > 1000$ ft, where excess power ps is defined by

$$ps = [T_{\max} - D(1.)]V/W \tag{18}$$

(These limits reflect the desire to maintain $\dot{E} > 0$ when ΔE is large and to limit the energy loss rate when ΔE is small or negative.) Equation (17) is an implicit equation for $\eta_{\max}$. An efficient solution method is to search for the smallest η such that

$$\dot{E}\,(T_{\max}, \eta) < f\,(\Delta E)$$

where $\eta = 1$, initially, and is increased in 1-g increments. Quadratic interpolation is then used to solve for $\eta_{\max}$ satisfying Eq. (17).

Energy Capture Flight-Path Guidance

Flight-Path Angle Control for Simultaneous Height/Speed Capture

When ΔE is large, thrust command is at one of the saturation limits and the altitude/speed dynamics can be partitioned into "slow" time scale dynamics where desired flight-path angle γ_r is the control, and "fast" time scale dynamics where vertical load factor is the control, and γ_r is a reference value to be tracked.

The model dynamics for the slow time scale are obtained from Eqs. (3), (4), and (12)

$$dh/dt = V \sin\gamma_r \tag{19}$$

$$\frac{d\gamma_r}{dt} = 0 = \frac{g}{V}(\eta_{Vr} - \cos\gamma_r) \tag{20}$$

$$\dot{E}\,(T_c, \eta_r) = V(\sin\gamma_r + \dot{V}/g) \tag{21}$$

where T_c denotes thrust command. ($T_c = T_{\max}$ if $\Delta E > 0$, and $T_c = T_{\min}$ if $\Delta E < 0$.) From Eqs. (20) and (5) we obtain

$$\eta_r = [(\eta_H)^2 + (\cos\gamma_r)^2]^{1/2} \tag{22}$$

For ΔE large, the energy rate in Eq. (21) is only a weak function of η_r since $\eta_{Vr} \approx 1$. This means that γ_r regulates the distribution of energy rate between height and speed, i.e., solving for $\sin\gamma_r$ such that $\dot{v} = 0$ in Eq. (21) gives maximum climb rate, whereas choosing $\gamma_r = 0$ gives maximum acceleration or speed rate.

Now, to obtain simultaneous height and speed capture, it suffices to choose γ_r such that time-to-height capture equals time-to-speed capture

$$t_f = \Delta h / \frac{dh}{dt} = \Delta V / \dot{V} \tag{23}$$

where

$$\Delta h = h_f - h, \qquad \Delta V = V_f - V$$

$$V_f = [2g\,(E_f - h_f)]^{1/2} \tag{24}$$

Substituting Eqs. (19), (22), and (23) into Eq. (21), and rearranging terms, yields an implicit equation for γ_r

$$\dot{E}\,(T_c, \eta_r)/V = [V\Delta V/(g\Delta h) + 1] \sin\gamma_r \tag{25}$$

Equation (25) becomes ill-conditioned as $|\Delta h|$ approaches zero. Consequently, γ_r is simply set to zero below a threshold $|\Delta h|$. Otherwise, a solution to Eq. (25) is attempted. An efficient solution method for $\Delta E > 0$ is to search at fixed increments in $|\gamma|$, beginning at $\gamma = 0$, until the left side Eq. (25) is first exceeded or a maximum $|\gamma_r|$ value is achieved. Quadratic interpolation is then used to find a precise solution for Eq. (25).

Inner-Loop Vertical Load Factor

Once γ_r has been determined, η_V is calculated to drive the flight-path angle γ to γ_r. In the inner loop or fast time scale, Eq. (20) is replaced with a second-order feedback control law

$$\frac{d_\gamma}{dt} = k_\gamma\,(\gamma_r - \gamma) + k_\eta\,(\eta_V - \cos_\gamma)\,g/V \tag{26}$$

where η_V is a lag filtered version of the vertical load-factor command η_{Vc}, and η_{Vc} is determined from Eq. (4)

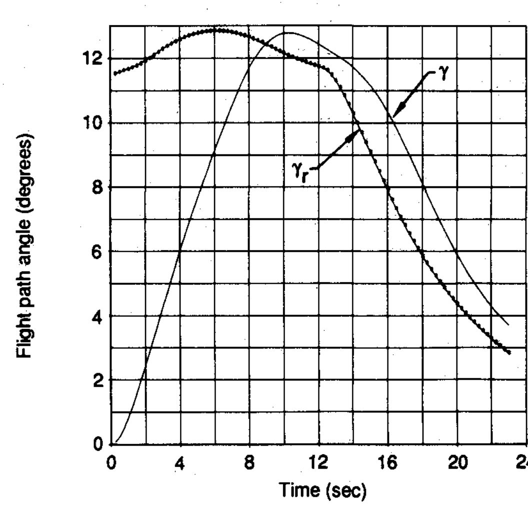

Fig. 2 Flight-path angle tracking for climbing turn scenario.

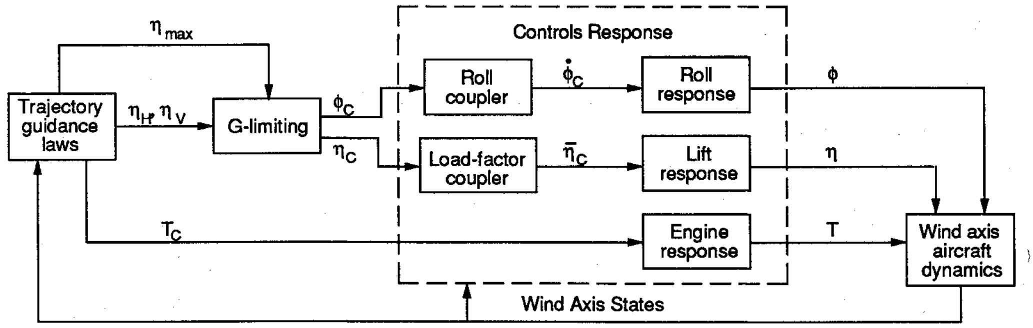

Fig. 3 Five-DOF flight simulation flow diagram.

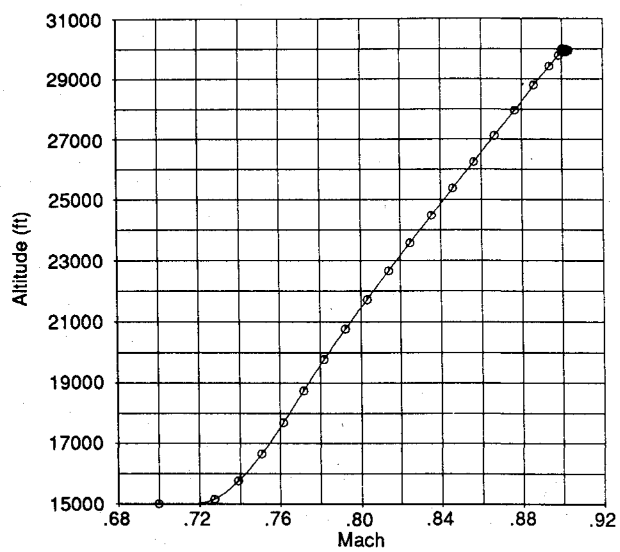

Fig. 4 Height/Mach profile for large energy transition.

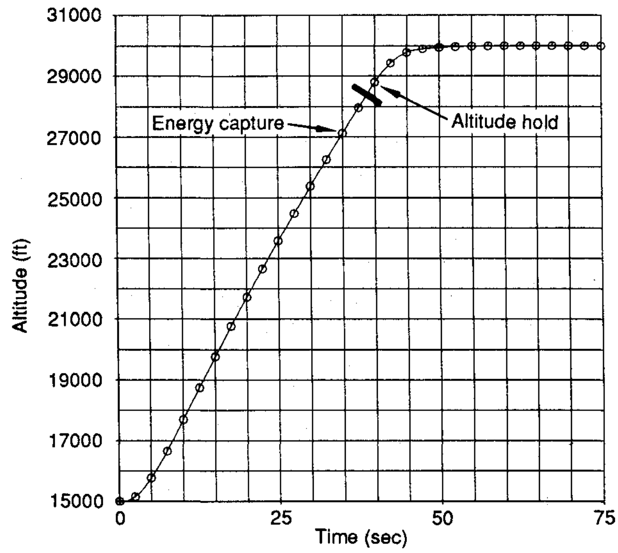

Fig. 5 Altitude transition for coordinated energy capture.

$$\frac{d\eta_V}{dt} = \frac{\eta_{Vc} - \eta_V}{\tau_v} \tag{27}$$

$$\eta_{Vc} = \cos\gamma + \frac{V}{g}\frac{d\gamma}{dt} \tag{28}$$

In the preceding equations [Eqs. (26–28)], k_γ and k_η are chosen to have specified closed-loop damping and bandwidth. Vertical load factor η_V is also subject to minimum and maximum limits as described earlier.

Typical results with the inner-loop controller are shown in Fig. 2. These results correspond to a constant speed climbing turn with $\Delta\psi = 180$ deg, $\Delta h = 3000$ ft, and $\Delta V = 0$. In this case, the actual flight path crosses the reference γ_r at 9 s and then follows the reference flight path with a lag error of about 2 s until energy capture is exited at time = 23 s. (The lag error in tracking the reference value is due to nonzero $d\gamma_r/dt$.)

Transition to Altitude/Energy Hold

When energy to go $|\Delta E| < 2000$ ft, the energy hold thrust control law is activated. However, the energy capture control law for η_V remains in effect until either the thrust command backs off of the saturation limit $T_{\min}$ or $T_{\max}$ or time-to-final altitude is less than a threshold value (5 s). The altitude hold control law is initiated when either of these conditions occur. The altitude/energy hold control laws then capture and maintain the desired cruise state.

Simulation Results

Five-Degree-of-Freedom Airplane Simulation

The energy management guidance laws were developed to be used along with horizontal plane guidance laws for three-dimensional aircraft flight control. A simple heading capture and hold guidance algorithm is used to illustrate cruise transition performance in the test cases below. A five-DOF model of a high-performance fighter aircraft, including lift, roll, and engine response dynamics, was used to evaluate the guidance laws. (The trajectory guidance laws are based on point-mass aircraft dynamics, but performance evaluations are performed with a five-DOF aircraft model with more realistic aircraft response dynamics and performance limitations. The model is considered a five-DOF simulation since yaw dynamics are ignored.)

Figure 3 is a top-level flow diagram of the five-DOF flight simulation. The vertical load factor and thrust commands η_V and T_c of the energy management laws and a horizontal load-factor command η_H from the heading hold law are the basic outputs of the guidance laws. The load-factor g limiting described earlier is actually performed on the load-factor commands so that total load factor is limited in magnitude, but the direction of inertial acceleration is unchanged. The outputs after g limiting are bank angle command and total load-factor command

$$\phi_c = \tan^{-1}(\eta_H/\eta_V) \tag{29}$$

$$\eta_c = (\eta^2_H + \eta^2_V)^{1/2} \tag{30}$$

These commands are transformed into aircraft response quantities roll ϕ and actual load factor η by independent roll and lift response channels. Engine response is simply modeled with a first-order lag and a rate limiter on delta thrust.

In the simulation, wind axis accelerations are computed and transformed to inertial axes for state propagation. The model is equivalent to using the following wind axis aircraft dynamics for a flat Earth coordinate system with thrust along the velocity vector[1]

$$\dot{X} = V \cos\psi \cdot \cos\gamma \tag{31}$$

$$\dot{Y} = V \sin\psi \cdot \cos\gamma \tag{32}$$

$$\dot{V} = g\ [(T \cos\alpha - D)/W - \sin\gamma] \tag{33}$$

$$\dot{\psi} = g\ [\eta + (T/W) \sin\alpha] \sin\phi/(V\cos\gamma) \tag{34}$$

$$\dot{h} = V \sin\gamma \tag{35}$$

$$\dot{\gamma} = g\ \{[\eta + (T/W) \sin\alpha] \cos\phi - \cos\gamma\}/V \tag{36}$$

In Eq. (33), the drag D is computed using Eq. (8), and angle of attack is similarly obtained from a table interpolation with independent variables Mach and C_L.

Vertical Energy Climb Test Case

Results with the energy capture guidance law are presented for two test cases. The first test case is a subsonic energy climb at constant heading. The aircraft is assumed cruising at 15,000 ft altitude and Mach = 0.7 at the start of the simulation and is requested to transition to 30,000 ft altitude and Mach = 0.9. The Mach/altitude profile for this transition (Fig. 4) shows nearly linear convergence to the final Mach and altitude states. The time history plots for altitude and energy, Figs. 5 and 6, show that the transitions are smooth with no significant transients when switching from energy capture to altitude/energy hold at t = 39 s. Moreover, altitude and energy capture of the desired final values occur simultaneously at about t = 50 s. The response characteristics for flight-path angle are shown in Fig. 7. Following an initial transition period, the flight path peaks at a 32-deg climb angle, decreases smoothly until altitude hold is initiated at 39 s, and then decreases rapidly to 2 deg at t = 47 s. When altitude hold is initiated, vertical load-factor command (see Fig. 8) becomes negative, commanding the aircraft to perform a 180 deg rollover for fast altitude convergence. (Negative vertical load factors are obtained by rolling the aircraft 180 deg and applying positive lift.) Finally, the thrust command profile in Fig. 9 shows a well-behaved stable response in transitioning from max afterburner to steady-state cruise.

It should be noted in this test case that energy capture by itself is not optimal in attaining the final end states in minimum time. A near minimum time energy climb guidance law using energy capture when $\Delta E < 8000$ ft was also applied to this test case. This algorithm was about 5 s faster in performing the transition because a more optimum height/Mach pro-

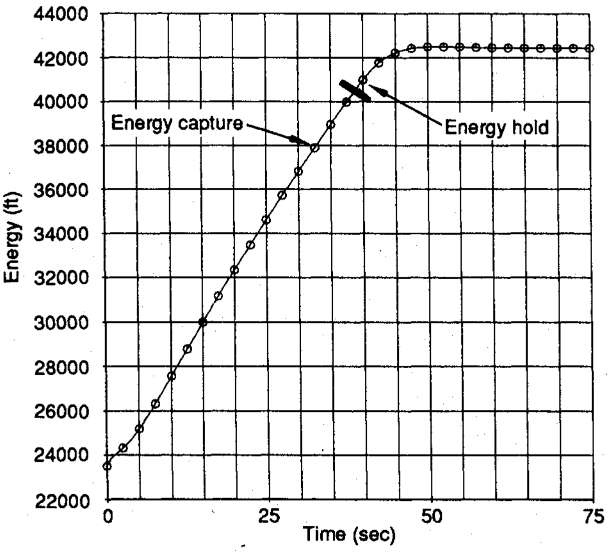

Fig. 6 Energy transition for coordinated altitude capture.

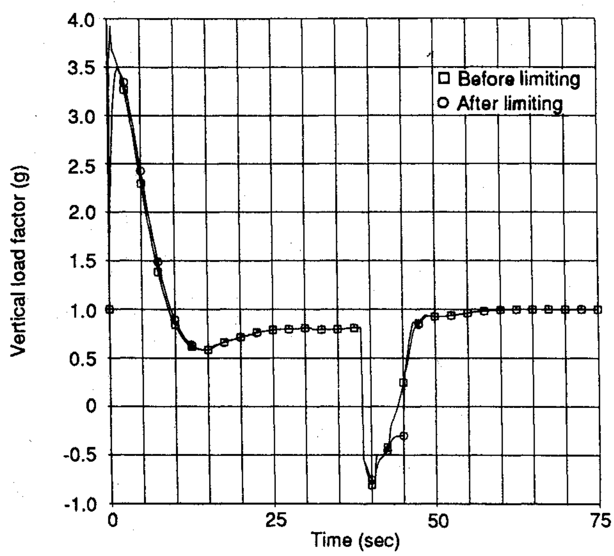

Fig. 8 Vertical load-factor guidance commands.

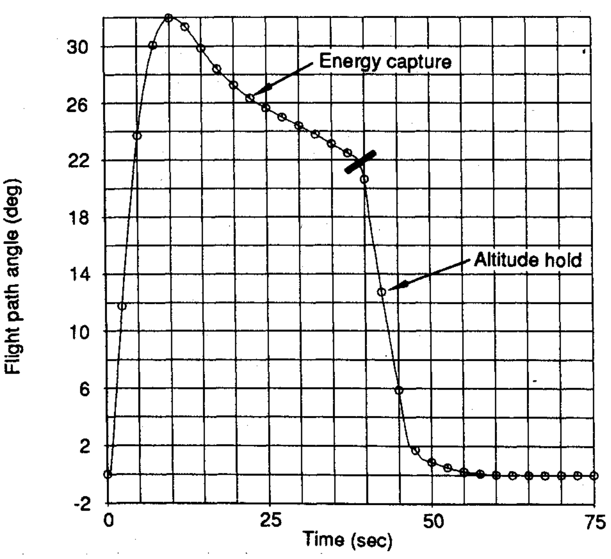

Fig. 7 Flight-path angle shows smooth guidance transitions.

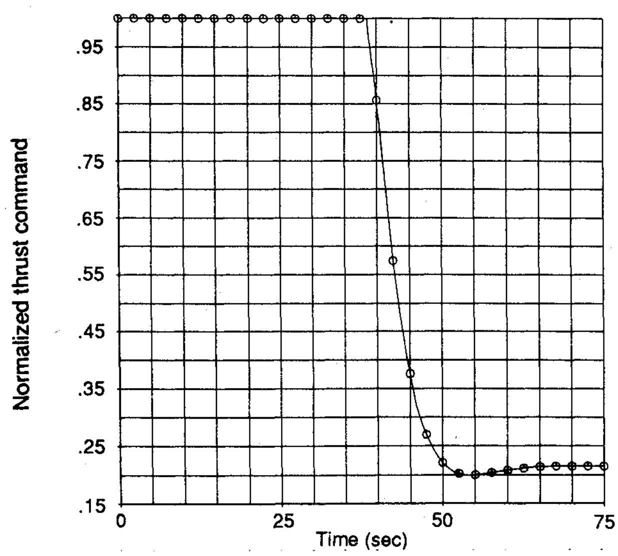

Fig. 9 Thrust commands for energy capture/hold.

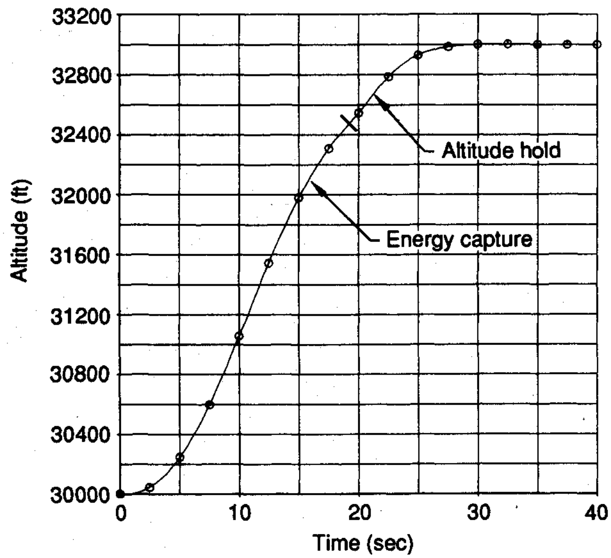

Fig. 10 Altitude transition for climbing turn.

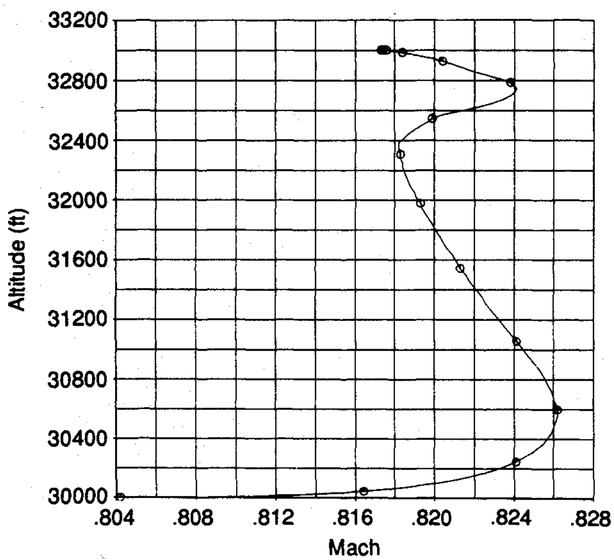

Fig. 11 Height/Mach profile shows coordinated approach to final cruise states.

file was used. However, for moderate energy climbs less than 10,000 ft, energy capture has proven to be nearly optimal in minimizing transition time.[7]

90-Deg Climbing Turn Test Case

The second test case is a constant speed climbing turn. The aircraft begins at steady-state cruise at 30,000 ft altitude and Mach = 0.8 and transitions 90 deg in heading and 3,000 ft in altitude, at constant speed. (Mach increases 0.017 due to the change in altitude.) This is a difficult test case for minimum time transitioning since the horizontal turn must be achieved simultaneously with the energy/height capture. The difficulty is that the horizontal and vertical guidance laws tend to ask for more lift than is consistent with maintaining the aircraft's energy state, even at maximum thrust. Without good energy management, the result is a significant loss in speed during the climb and turn transition, ending with a high thrust acceleration to regain lost speed.

The results using energy capture for this case are summarized in Figs. 10 and 11. Both the altitude transition shown in Fig. 10 and the energy transition are smooth and achieve convergence by $t = 28$ s. Moreover, the Mach/altitude profile shown in Fig. 11 reveals that, after an initial Mach transient, the energy capture/energy hold guidance laws maintain Mach within 0.01 during the subsequent state transition, i.e., the guidance laws maintained speed throughout the transition. Additional details on the control interactions for this case were presented in an earlier version of this paper.[8]

Conclusions

This paper introduces a time-to-go matching principle for near-minimum-time energy state transitions. The energy management guidance laws discussed use throttle regulation to control total energy state and flight-path angle to coordinate height/speed transitions. The advantages of this methodology are 1) precise controls coordination for nearly simultaneous (within 2 s) height and speed capture of the desired cruise point, and 2) energy conservative, near-minimum-time transitions for energy-to-go less than 10,000 ft. These guidance laws can be combined with horizontal plane guidance laws for three-dimensional maneuvering and cruise state transitions.

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